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A LETTER TO Dr. *CHEYNE*,

CONTAINING

An Account of the Motion of Water through Orifices and Pipes;

And an ANSWER to

Dr. MORGAN's Remarks

ON

Dr. ROBINSON's Treatiſe

OF THE

Animal Oeconomy.

DUBLIN:
Printed by S. POWELL,
For G. EWING at the *Angel* and *Bible*,
and W. SMITH at the *Hercules*,
Bookſellers in *Dame*'s-*ſtreet*, 1735.

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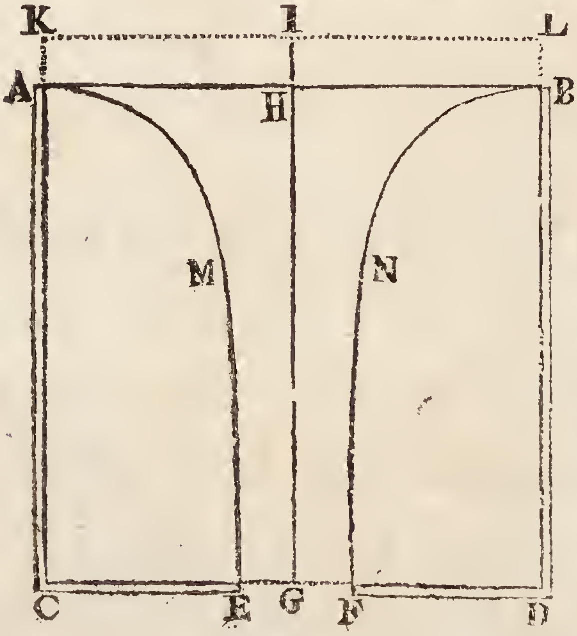
A LETTER TO Dr. *CHEYNE*, &c.

SIR,

KNOWING you to be a Perſon of Candour and Judgment, and throughly acquainted with Philoſophical Subjects, I take the Liberty to ſend you this Paper, which contains an Account of the Motion of Water through Orifices and Pipes, and a Vindication of my Treatiſe of the *Animal Oeconomy* from certain Objections offered againſt it by Dr. *Morgan*, in his late Book, intitled, *The mechanical Practice of Phyſick.*

PROPOSITION I.

IF A C D B *be a cylindrical Veſſel filled with Water,* A B *its upper Orifice,* C D *its Bottom parallel to the Horizon,* E F *a circular Hole in the middle of the Bottom,* G H *the Axis of the Cylinder perpendicular to the Horizon,* G I *the Axis produced till* I H *becomes equal to the Space thro' which a hea-vy*



vy Body muſt deſcend in vacuo *to acquire a Velocity equal to the Velocity of the Water in the Surface* A B; *if* A *be put for the Area of the Surface of the Water* A B, a *for the Area of the Hole* E F, H *for* G H *the perpendicular Height of the Water in the Veſſel above the Hole,* V *for the Velocity of the Water flowing through the Hole, and* v *for the Velocity of the Water in the Surface* A B; *and laſtly, if the Veſſel be ſuppoſed to be kept conſtantly full, by being ſupply'd at the Top as faſt as the Water runs out through the Hole, and the Water deſcend from the Top of the Veſſel to the Hole freely and without Reſiſtance; I ſay that* V *will be equal to the Velocity acquired in falling* in vacuo *through the Space* $\frac{V^2 H}{V^2 - v^2}$, or $\frac{A^2 H}{A^2 - a^2}$, or $\frac{IG \times H}{IG - IH}$, or IG.

For IH being the Space through which a heavy Body muſt fall *in vacuo*, to acquire a Velocity equal to the Velocity of the Water in the Surface A B, and the Water being ſuppoſed to deſcend from the Surface to the Hole freely and without Reſiſtance, IG will be equal to the Space through which a heavy Body muſt fall *in vacuo* to acquire the Velocity of the Water in the Hole E F: But the Velocities acquired by a Body falling *in vacuo* through the Spaces I G and I H are in the ſubduplicate Ratio of thoſe Spaces, on Suppoſition that the Gravity of the Body is the ſame in I as in G, as it will be without any ſenſible Error if the Point I be but at a ſmall Diſtance from the Surface of the Earth; and the ſame Velocities V and v, are alſo as the Areas A and a, becauſe equal Quantities of Water paſs through the Surface A B and Hole

EF in the ſame Time; and therefore $V . v :: A . a :: \sqrt{IG} . \sqrt{IH}$; and by Squaring, and Diviſion of Proportion, $\frac{V^2}{V^2-v^2}=\frac{A^2}{A^2-a^2}=\frac{IG}{IG-IH}=\frac{IG}{H}$, and, by multiplying by H, $\frac{V^2H}{V^2-v^2}=\frac{A^2H}{A^2-a^2}=\frac{IG\times H}{IG-IH}=\frac{IG\times H}{H}=IG$: And therefore V will be equal to the Velocity acquired by falling *in vacuo* through the Space $\frac{V^2H}{V^2-v^2}$, or $\frac{A^2H}{A^2-a^2}$, or $\frac{IG\times H}{IG-IH}$, or IG. Which was to be proved.

Cor. 1. H, the perpendicular Height of the Water in the Veſſel, is $=IG\times\frac{\overline{V^2-v^2}}{V^2}=IG\times\frac{\overline{A^2-a^2}}{A^2}$.

Cor. 2. V, the Velocity with which the Water flows through the

Hole, is as $\sqrt{\frac{V^2 H}{V^2 - v^2}}$, or as $\sqrt{\frac{A^2 H}{A^2 - a^2}}$, or as $\sqrt{\frac{IG \times H}{IG - IH}}$, or as $\sqrt{IG}$.

Cor. 3. If a be equal to A, V will be nothing. For when a is equal to A, v will be equal to V by this *Propoſition*: But when a=A, and v=V, H will be nothing by *Cor.* 1. And conſequently V will be nothing by *Cor.* 2.

The Truth of this *Corollary* will appear likewiſe, by conſidering that when a is equal to A, I H will be equal to I G, and the Point H will coincide with the Point G: But when the Point H coincides with the Point G, the perpendicular Height of the Water in the Veſſel will be deſtroy'd, and when the perpendicular Height of the Water in the Veſſel is deſtroy'd, there can be no Velocity: And therefore when $a=A$, V will be nothing.

Farther, to ſuppoſe a to be equal to A, and conſequently v to be equal to V, will from the Nature of Gravity make V nothing. For Gravity accelerates the Motion of the Water from the Surface to the Hole, and makes the Velocity through the Hole greater than at the Surface, while there is the leaſt perpendicular Diſtance between them; and therefore to make the two Velocities equal, will be to deſtroy the perpendicular Height of the Water in the Veſſel: But when the perpendicular Height of the Water is deſtroy'd, there can be no Velocity: And therefore to ſuppoſe v to be equal to V, will be to make V nothing.

From all this it appears, that the Velocities v and V can never become equal, but in the Inſtant of their Evaneſcence on an infinite Di-

Diminution of the perpendicular Height of the Water in the Veſſel, and a conſequent Coincidence of the Points I and H with the Point G.

Cor. 4. If A be greater than A, H will be negative, and V will be affirmative. For when a is greater than A, $A^2 - a^2$ will be negative, and conſequently H will be negative by *Cor.* 1. And, when $A^2 - a^2$ and H are both negative, V, the Velocity with which the Water flows through the Hole, will be affirmative by *Cor.* 2.

A negative perpendicular Height of the Water in the Veſſel, and an affirmative Velocity neceſſarily require an Inverſion of the Veſſel, or turning of its Bottom upwards, by which Inverſion the Hole will become the upper Orifice and the upper Orifice become the Hole, a will become A, and A will become a, v will become V, and V become v, and the Velocity will be affirmative, that is, the Water will move downwards, as it ought to do from the Nature of Gravity. That an Inverſion of the Veſſel is neceſſary, will appear farther by the following Argument. When a is greater than A, the Veſſel will be conical with its wider End downwards: But, from the Nature of Gravity, Water poured in at the Top or narrower End of ſuch a Veſſel, will deſcend in a cylindrical Column, which cannot fill the Baſe, as this *Propoſition* requires: And therefore there muſt be an Inverſion of the Veſſel, and ſuch a Change of Symbols as I have mentioned.

Cor. 5. If the Hole be exceeding ſmall in compariſon of the upper Orifice, the Velocity with which the Water flows through the Hole, will, without any ſenſible Error, be equal to the Velocity which a heavy Body will acquire in falling *in vacuo* through a Space equal to the perpendicular Height of the Water in the Veſſel, that is, V will, without any ſenſible Error, be equal to the Velocity acquired in falling *in vacuo* through a Space equal to H. For when a is exceeding ſmall in compariſon of A, $A^2 - a^2$ will be very nearly equal to A^2, and $\frac{A^2H}{A^2 - a^2}$ will be very nearly equal to H: But, by this *Propoſition*, V is equal to the Velocity acquired in falling *in vacuo* through a Space equal to $\frac{A^2H}{A^2 - a^2}$: And therefore V will be very nearly equal to the Velocity acquired in falling *in vacuo* through a Space equal to H.

Cor. 6. The Force which can generate the whole Motion of the Water flowing out thro' the Hole, is equal to the Weight of a Cylinder of Water whoſe Baſe is the Hole, and whoſe Altitude is twice the Space through which a heavy Body muſt fall *in vacuo* to acquire the Velocity with which the Water flows through the Hole; that is, the Force is equal to the Weight of the Cylinder of Water $a \times 2$ I G. For the Time, in which the Water flowing out through the Hole becomes equal to the Cylinder of Water $a \times 2$ I G, is equal to the Time in which that Cylinder of Water in falling *in vacuo* by the conſtant Action of its own Weight will acquire a Velocity equal to that with which the Water flows out, that is, a Velocity equal to V; and from an Equality of the Times of the Motions, of the Quantities of Matter moved, and of the Velocities, the Motions of the effluent Water and of the deſcending Cylinder will be equal: But when two Motions are equal, the Forces generating thoſe Motions are equal: And conſequently, the Force which can generate the whole Motion of the Water flowing out through the Hole, is equal to the Weight of a Cylinder of Water whoſe Magnitude is $a \times 2\,IG$.

SCHOLIUM.

Since, by this *Propoſition*, the Velocity with which the Water flows through the Hole, is equal to the Velocity which a Body will acquire by falling *in vacuo* through a Space equal to $\frac{A^2H}{A^2-a^2}$, on Suppoſition that the Water deſcends down the Veſſel freely and without Reſiſtance; we may, by knowing the Velocity acquired by a heavy Body in falling through ſuch a Space, find the Velocity with which the Water flows out through the Hole. For, according to Sir *Iſaac Newton*, a Body falling *in vacuo* near the Surface of the Earth will deſcribe $193\frac{1}{3}$ Inches or $16\frac{1}{9}$ Feet in one Second Minute of Time, and will have acquired a Velocity in the Time of the Fall, which being continued uniform would make it deſcribe twice that Space, that is $386\frac{2}{3}$ Inches or $32\frac{2}{9}$ Feet in one Second: But uniform Velocities are as the Spaces deſcribed by them in one Second, and the Velocities acquired in falling *in vacuo* through the Spaces $16\frac{1}{9}$ and $\frac{A^2H}{A^2-a^2}$ are in the ſubduplicate Ratio of thoſe Spaces: And therefore $32\frac{2}{9} . \sqrt{16\frac{1}{9}} :: V . \sqrt{\frac{A^2H}{A^2-a^2}}$; whence V =

8.02773

$8.02773\sqrt{\frac{A^2H}{A^2-a^2}}$ Feet $= 96.33276$ $\sqrt{\frac{A^2H}{A^2-a^2}}$ Inches. This is the Velocity with which the Water flows through the Hole, ſetting aſide the Reſiſtance of the Air, and ſuppoſing the Water to deſcend down the Veſſel freely and without Reſiſtance.

The true Velocity with which the Water flows out through the Hole, will be had by applying the Quantity of Water diſcharged by Experiment, to the Area of the Hole and Time of the Diſcharge taken together; that is, putting Q for the Quantity of the Diſcharge in cubick Inches, D for the Diameter of the Hole in Inches or Parts of an Inch, and T for the Time of the Diſcharge in Seconds, by $\frac{Q}{.785816D^2T}$.

Now if the Water deſcend freely and without Reſiſtance, this Meaſure of the Velocity will be equal to the former, that is, $\frac{Q}{.7853816D^2T} = 96.33276\sqrt{\frac{A^2H}{A^2-a^2}}$; whence $Q = 75.65957D^2T\sqrt{\frac{A^2H}{A^2-a^2}}$, and, putting W for the Weight of this Bulk of Water in Ounces *Troy*, $W = 39.93144D^2T\sqrt{\frac{A^2H}{A^2-a^2}}$. But it has been found by Experiments, that the Bulk and Weight of Water diſcharged, always fall conſiderably ſhort of what they ought to be by theſe Rules; Sir *Iſaac Newton* found them to be leſs in the Proportion of 441 to 625, or of 1 to $\sqrt{2}$ nearly: And therefore Q and W muſt both be leſſened in this Proportion, and then we ſhall have $Q = 67.97239\sqrt{\frac{A^2H}{A^2-a^2}}$, and $W = 53.38426D^2T\sqrt{\frac{A^2H}{A^2-a^2}}$.

That leſs Water flows through the Hole in a given Time, than would flow through it if the Water deſcended down the Veſſel without any Reſiſtance, muſt be owing either to the Water's not filling the Hole when it paſſes through it, or to its paſſing through it with a leſs Velocity than that which is required in falling *in vacuo* through a Space equal to $\frac{A^2H}{A^2-a^2}$: But by carefully obſerving the Water as it flows out, I find that it fills the Hole if the Plate in which the Hole is made be very thin, and if the Plate be thick and the Hole be a ſhort Pipe, that it fills the inner Orifice of the Pipe which is contiguous to the Water in the Veſſel, though it does not fill the outer Orifice on account of the Contraction of the Vein, which Contraction extends from the inner Orifice of the Pipe, which is to be conſidered as the Hole, to about the Diſtance of its Diameter: And therefore the Water paſſes through the Hole with a leſs Velocity than that which is acquired in falling *in vacuo* through a Space equal to $\frac{A^2H}{A^2-a^2}$. It paſſes through the Hole with a leſs Velocity, from its not deſcending freely and without Reſiſtance, and it does not deſcend freely and without Reſiſtance from its moving laterally as well as downwards throughout its Paſſage from the upper Orifice to the Hole. The lateral Motion of the Water, or the flowing together of its Parts from all Sides of the Veſſel throughout its whole Deſcent, gives a Reſiſtance to its Motion downwards, and that Reſiſtance leſſens the perpendicular Velocity, and makes the Quantity of Water diſcharged in a given Time leſs than it would be if there was no ſuch lateral Motion, but the Water deſcended only perpendicularly from A B to E F.

The Velocity therefore with which the Water flows through the Hole, is equal to the Velocity which a heavy Body would acquire in falling *in vacuo* through a Space equal to one half of the Space $\frac{A^2H}{A^2-a^2}$. For the Velocity with which the Water flows through the Hole, is to the Velocity acquired in falling *in vacuo* through the Space $\frac{A^2H}{A^2-a^2}$ as 1 to $\sqrt{2}$, from Sir *Iſaac Newton*'s Experiments: But from the Law of the Deſcent of heavy Bodies *in vacuo*, the Velocity acquired in falling through one half of the Space $\frac{A^2H}{A^2-a^2}$ is to the Velocity acquired in falling through the whole Space $\frac{A^2H}{A^2-a^2}$ as 1 to $\sqrt{2}$: And therefore the Velocity with which the Water flows thro' the Hole is equal to the Velocity acquired by falling *in vacuo* through one half of the Space $\frac{A^2 H}{A^2 - a^2}$.

The true Quantity of Water diſcharged will likewiſe be had by ſuppoſing the Velocity with which the Water flows through the Hole to be equal to that which a heavy Body will acquire in falling in *vacuo* through the Space $\frac{A^2 H}{A^2 - a^2}$, and the Hole to be contracted and its Area leſſened in the Proportion of 441 to 625 or 1 to $\sqrt{2}$. For, ſince the Quantity of Water diſcharged is as the Velocity with which the Water flows through the Hole, the Area of the Hole, and Time of the Motion taken together, it is evident that the ſame Quantity of Water will be diſcharged in a given Time, whether the Velocity acquired in falling *in vacuo* through the Space $\frac{A^2H}{A^2-a^2}$, or a the Area of the Hole, be leſſened in the Proportion of 1 to $\sqrt{2}$. Sir *Iſaac Newton* found the Area of a tranſverſe Section of the Vein at the Diſtance of about a Diameter from the Hole, to be leſs than the Area of the Hole in the ſaid Proportion. And in the Determination of this Motion he ſuppoſes the Hole to be contracted in that Proportion, and the Velocity, with which the Water flows through it, to be equal to the Velocity in the contracted Part of the Vein, that is, equal to the Velocity which a heavy Body will acquire in falling *in vacuo* through G I, or $\frac{A^2H}{A^2-a^2}$.

In order to know, whether the Velocities of Water flowing through circular Holes of different Diameters at the ſame perpendicular Diſtance from the Surface of the Water, be all equal, and what Relation the Velocity of Water flowing through a Hole bears to the Velocity of Water flowing through a Pipe of an equal Diameter and at an equal perpendicular Diſtance from the Surface of the Water, I got Mr. *Stokes*, a skilful and accurate Mathematical Inſtrument-maker in this City, to make a very exact *Apparatus* for aſcertaining theſe Things by Experiments. And from the Experiments I compoſed the two following Tables.

The firſt Table contains, in the firſt Column the Time of the Diſcharge in Seconds, in the ſecond the perpendicular Heights of the Water above the Hole in *London* Feet, in the third the Diameters of the Holes in Parts of an Inch, in the fourth the Weights of Water diſcharged in Ounces *Troy*, and in the

TABLE I.

T	H	D	V	$\frac{V}{D^2}$
5″	4	$\frac{2}{10}$	13.267	3317
		$\frac{4}{10}$	49.	3062
		$\frac{5}{10}$	76.	3040
		$\frac{8}{10}$	186.	2906
	2	$\frac{2}{10}$	9.	2250
		$\frac{4}{10}$	35.	2193
		$\frac{5}{10}$	54.	2160
		$\frac{8}{10}$	133.75	2090

the fifth the Velocities meaſured by the Weights of Water diſcharged apply'd to the Squares of the Diameters of the Holes.

By this Table, the Velocities of Water flowing through Holes of different Diameters, and at the ſame perpendicular Diſtance from the Surface of the Water, are all nearly equal; only the Velocity is ever ſomething greater through a ſmaller Hole than through a larger.

The ſecond Table conſiſts of three Parts, and each Part of three Columns. The firſt Column of each Part contains the Diameter of

Table II.

D	L	V	D	L	V	D	L	V
$\frac{2}{10}$	O	13.267	$\frac{4}{10}$	O	49.	$\frac{8}{10}$	O	186.
	D	14.985		D	57.		D	213.25
	2 D	15.		2 D	58.5		2 D	234.
	3 D	14.367		3 D	55.		3 D	226.5
	4 D	14.300		4 D	54.5		4 D	221.
	5 D	14.233		5 D	53.75		5 D	212.
	10 D	13.212		16 D	49.		16 D	196.
							23 D	186.

the Pipe in Parts of an Inch, the ſecond the Lengths of the Pipes beginning from O, that is from an Hole, and the third the Velocities expreſſed by the Weights of Water diſcharged in five Seconds of Time. The Holes and Pipes were all at the

Diſtance of 4 Feet from the Surface of the Water, and the Pipes lay all parallel to the Horizon.

By this Table, the Velocity increaſes from the Hole till the Length of the Pipe becomes equal to about twice its Diameter, that is, equal to about 2 D, and is greater there than at any other Length of the Pipe. The greateſt Velocities in theſe Pipes in Proportion to the Velocities through their reſpective Holes, which Holes may be conſidered as Pipes of infinitely ſmall Lengths, are as the Numbers 1130, 1215 and 1258, in Proportion to 1000: Whence we learn, that the greateſt Velocity in a Pipe in Proportion to the Velocity through an Hole of an equal Diameter and at an equal perpendicular Diſtance from the Surface of the Water, is ſomething greater in a wider Pipe than in a narrower one.

From the Length 2 D, the Velocity leſſens continually on increaſing the Length of the Pipe, and becomes equal to the Velocity thro' the Hole, when the Length of the Pipe becomes equal to 25.71478 $D\sqrt{D}$. For the Velocities through the Pipes were nearly equal to the Velocities through their reſpective Holes, when the Lengths of the Pipes were 10 D, 16 D, and 23 D, that is, 2, 6.4, and 18.4 Inches: But 2, 6.4, and 18.4, are nearly in the ſeſquiplicate Ratios of the Diameters $\frac{2}{10}$, $\frac{4}{10}$, and $\frac{8}{10}$: And therefore $18.4 . \frac{8}{10} \times \sqrt{\frac{8}{10}} :: L . D\sqrt{D}$; whence $L = 25.71478\, D\sqrt{D}$ nearly.

From five times that Length of a Pipe forward at which the Velocity is equal to the Velocity through an Hole of an equal Diameter, that is, from the Length 128.5739 $D\sqrt{D}$ forward, the Velocity will be meaſured nearly by the inverſe ſubduplicate Ratio of the Length of the Pipe; V will be nearly as $\frac{1}{\sqrt{L}}$. For, by the firſt Experiment, *Anim. Oecon.* p. 29, the Velocities in two Pipes, whoſe Lengths were 2 Feet and 8 Feet, and whoſe common Diameter was $\frac{345}{1000}$ Parts of an Inch, were nearly in the reciprocal ſubduplicate Ratios of the Lengths of the Pipes: But 128.5739 $D\sqrt{D}$ is nearly equal to 2 Feet, the Length of the ſhorter Pipe: And therefore from the Length 128.5739 $D\sqrt{D}$ forward, the Velocity will be meaſured nearly by the reciprocal ſubduplicate Ratio of the Length of the Pipe; V will be nearly as $\frac{1}{\sqrt{L}}$. Hencc it follows, that from the Length 128.5739 $D\sqrt{D}$ forward, the Rectangle under the Velocity and ſquare Root of the Length of the Pipe will be given; that is, $V\sqrt{L}$ will be given, or be the ſame in Pipes of Lengths greater than 128.5739 $D\sqrt{D}$. It was nearly ſo in Pipes of half an Inch in Diameter, and of different Lengths from a Pipe of 4 Feet in Length to one of 100, when the perpendicular Height of the Water in the Veſſel was 3 Feet. From the Nature of the Motion of Water through Pipes, I think there muſt be a certain Length of a Pipe of a given Diameter beyond which $\frac{1}{\sqrt{L}}$ does not meaſure the Velocity; but what that Length is I cannot ſay for want of Experiments.

The Reaſon why this Meaſure of the Velocity does not begin to obtain till the Pipe be of a certain Length, may be this. The lateral Motion of the Water deſcending in the Veſſel, which Motion has been ſhewn to affect and diſturb the Motion of Water flowing through a

Hole, may likewiſe affect and diſturb its Motion through a Pipe, and hinder $\frac{1}{\sqrt{L}}$ from being an accurate Meaſure of the Velocity till the Pipe comes to be of ſuch a Length, that the Reſiſtance ariſing from the Weight of Water in the Pipe and from the internal Surface of the Pipe can in a good meaſure correct this Diſturbance; that is, till it comes to be of the Length $128.5739\ D\sqrt{D}$. And if this be the Reaſon why this Meaſure of the Velocity does not obtain with any Accuracy, till the Pipe comes to be of the Length $128.5739\ D\sqrt{D}$; then the Velocity with reſpect to $\frac{1}{\sqrt{L}}$ in Pipes exceeding this Length, will be always ſomething greater in a longer Pipe than in a ſhorter one (becauſe the Correction of the Diſturbance is greater in the former than in the latter)

latter) as I have always found it to be by Experiments.

This may ſuffice concerning the Motions of Water through Orifices and Pipes. I ſhall now proceed to Dr. *Morgan's* REMARKS, and ſhall ſhew that they have all been occaſioned by his not having duly attended to what Sir *Iſaac* and I delivered concerning theſe Motions.

In p. 68. l. 4, 5. the Doctor ſays that F *will ever be as* $D^2 H$, whereas had he attended to *Prop.* 36. *lib.* 2. *Newton.* he would have ſeen that F will never be as $D^2 H$, but when the Area of the Hole is infinitely little in compariſon of the Area of the Surface of the Water in the Veſſel, and the Pipe lies parallel to the Horizon. For I have ſhewn from that *Propoſition*, that the Force which can generate the Motion of Water flowing through a Hole, is equal to the Weight of a Cylinder of Water whoſe Magnitude is $a \times IG$, or $a \times \frac{A^2H}{A^2 - a^2}$: But this Force is evidently equal to the Force, which generates the Motion of Water flowing thro' a Pipe lying parallel to the Horizon, of an equal Diameter with the Hole, and inſerted into the inſide of the Veſſel at an equal perpendicular Diſtance from the Surface of the Water.

1. The Doctor ſuppoſes D and H to be given, and conſequently the moving Force, which is as D^2H, to be given; in which Caſe V will be meaſured by $\frac{1}{\sqrt{L}}$; and then affirms, *that if* L *be infinitely ſmall,* V *muſt be infinitely great, and if* L *be infinitely great,* V *will be infinitely ſmall.* Here I muſt acquaint this Author, that it by no means follows from L being infinitely ſmall, that V muſt

V muſt be infinitely great, any more than it follows from a Body's being ing diviſible *in infinitum*, that the Magnitude of that Body muſt be infinite. A finite Body will be infinitely great with reſpect to any of its Particles, when it is divided or ſuppoſed to be divided *in infinitum*; and a finite Velocity of Water flowing through a Hole, which may be conſidered as an infinitely ſhort Pipe, will be infinitely great with reſpect to the Velocity of Water flowing through a Pipe infinitely long: And yet it is evident that neither the Body nor the Velocity through the Hole is infinite. Small and great, infinitely ſmall and infinitely great, are Terms including a Compariſon of Things with one another; thus a Thing is called ſmall in compariſon of a Thing of the ſame Kind which is great, and infinitely ſmall in compariſon of a

Thing which is infinitely great; a Hole is a Pipe of an infinitely ſmall Length, in compariſon of a Pipe infinitely long. And therefore, ſince V is as $\frac{1}{\sqrt{L}}$, the Velocity of Water flowing thro' an Hole will be infinitely greater than the Velocity of Water flowing thro' an infinitely long Pipe, tho' in reality the Velocity thro' the Hole be finite, and no greater than the Velocity acquired in falling *in vacuo* thro' $\frac{A^2H}{A^2-a^2}$.

2. The Doctor, p. 69, ſuppoſes D *to be given, and* H *to be proportional to* L; *in which Caſe the Velocity thro' the Pipe will be given.* However ſtrange and abſurd this may appear to this Gentleman, I can aſſure him I have found it to be true by Experiments. Two Pipes of the ſame Diameter, whoſe

Lengths were 8 Feet and 2 Feet, placed at the Diſtances of 4 Feet and 1 Foot from the Surface of the Water, diſcharged 97 and 83 *Troy* Ounces of Water in half a Minute. The Quantity diſcharged by the ſhorter Pipe was ſomething leſs than the Quantity diſcharged by the longer, which was owing to the Diſturbance ariſing from the lateral Motion of the Water deſcending in the Veſſel, which is ever greater in a ſhorter Pipe than in a longer, and at a leſs Diſtance from the Surface of the Water than at a greater.

3. In his third Conſequence, p. 70, drawn from V being as $\sqrt{\frac{DH}{L}}$, this Author ſuppoſes L *to be given, and* D *to be as* $\frac{1}{H}$; *in which Caſe the Velocity will be given.* He thinks this very abſurd, as I gather from his fifth Conſequence, in which he

ſays

ſays expreſly, *that* D *and* L *have nothing to do in the Matter, and cannot alter the Velocities at all.* But in this he is greatly miſtaken; for both the Diameter and Length of a Pipe affect the Motion of Water moving thro' it, and are neceſſarily to be taken into the Meaſure of the Velocity, as fully appears from the firſt and ſecond Experiments in the Proof of *Prop.* 1. *Anim. Oecon.* And I ſhall now ſhew by an Experiment, that the Velocity of Water flowing thro' a Pipe of a given Length is nearly the ſame, when the Diameter of the Pipe is inverſly as its perpendicular Diſtance from the Surface of the Water; that is, V will be given, when D is as $\frac{1}{H}$. Two Pipes, of the ſame Length, whoſe Diameters were 1 and 2, placed at the Diſtances of 2 and 1 from the Surface of the Water, diſcharged 68 and

20 Ounces of Water in half a Minute. The Velocities from theſe Diſcharges were as the Numbers 17 and 20. The firſt Velocity was ſomething leſs than the ſecond; becauſe the firſt Pipe was wider and nearer to the Surface of the Water than the ſecond, and conſequently the Diſturbance given to the Motion thro' the firſt, was greater than the Diſturbance given to the Motion thro' the ſecond.

4. In his fourth Conſequence, p. 70, he ſuppoſes H *and* L *to be given; in which Caſe the Velocity will be as* $\surd$D. However abſurd this Conſequence may appear to the Doctor, I have proved it true by the ſecond Experiment in the Proof of *Prop.* 1. *Anim. Oecon.* to which I refer him.

5. In his fifth and laſt Conſequence, p. 70, he ſuppoſes D *and* L *to be given; in which Caſe* V *will be as* $\surd$H. This he allows to be

true, and ſays *it is the true Law of accelerating Gravity and Preſſure as determined by* Newton. But in this he is miſtaken. For V is not as $\sqrt{H}$ according to *Newton*, but as $\sqrt{\frac{A^2H}{A^2-a^2}}$, by *Cor.* 2. of the foregoing *Propoſition.*

The Doctor proceeds to ſhew the Abſurdity of what I aſſerted in my *Animal Oeconomy* from Sir *Iſaac Newton*, and have demonſtrated in the foregoing *Propoſition*, namely, that the Velocity of Water flowing thro' a Hole is equal to the Velocity acquired by a heavy Body in falling *in vacuo* through the Space $\frac{A^2H}{A^2-a^2}$.

1. *If this be the Space*, p. 71, 72, *through which a heavy Body muſt fall* in vacuo *to acquire a Velocity equal to the Velocity with which the Water flows through the Hole, then* V *will be as* $\sqrt{\frac{A^2H}{A^2-a^2}}$. *Now upon*

this Suppoſition, let the differential Quantity A^2-a^2 *be infinitely ſmall, or let* a *be only not equal to* A, *and then the Expreſſion* $\frac{A^2H}{A^2-a^2}$ *will be infinitely great; and conſequently the Space deſcribed, as well as the Velocity acquired, will be infinite, where the perpendicular Preſſure or Height of the Fluid is only finite and given; or which is the ſame thing, the Spaces deſcribed and the Velocities acquired, will be finite and infinite at the ſame Time and from the ſame Cauſes.* Here again the Doctor is greatly miſtaken. For by the third *Corollary* of the preceding *Propoſition*, (which *Propoſition* is a plain and obvious Conſequence of *Prop.* 36. *lib.* 2. *Newton.*) when a is equal to A, the Space deſcribed and Velocity are ſo far from being infinite, that both will be nothing. His Miſtake here aroſe from his not knowing that when a is equal to A, H will be nothing, by *Cor.* 1.

2. The Doctor goes on to ſhew the Abſurdity of my Aſſertion in the following Words, p. 72. *Let the differential* $A^2—a^2$ *be infinitely ſmall but negative, and then the Expreſſion* $\frac{A^2H}{A^2—a^2}$ *will be negative, but infinite ſtill; the Conſequence of which muſt be, that the Fluid in this Caſe cannot deſcend and flow thro' the Orifice* a *at all; but on the contrary muſt aſcend perpendicularly, and flow upwards with an infinite Velocity, in a Direction quite contrary to that of Gravity.* Here likewiſe the Doctor is miſtaken. For when a is greater than A, H will be negative, and V affirmative, by *Cor.* 4. And when H is negative the Veſſel muſt be inverted, or turned with its Bottom upwards, which being done this Objection will vaniſh, by *Cor.* 4.

Having ſhewn the Weakneſs of the Doctor's *Remarks* on my firſt *Propoſition*, I ſhall beg leave, before I anſwer his *Remarks* on my 24th *Propoſition*, to give a Demonſtration of the Law of the Blood's Motion, exhibited in *Prop.* 12, independent of my firſt Section concerning the Motion of Fluids thro' cylindrical Pipes; becauſe this Author ſeems to think, that all the mathematical Part of my Book depends on the Truth of that Section.

PROPOSITION II.

IF two healthful and perfectly well-proportion'd Bodies be ſituated alike with reſpect to the Horizon, if their Hearts be free from the Influences of all diſturbing Cauſes, and the Capacities of the two correſponding Ventricles

tricles be proportional to the Capacities of the two whole Syſtems of Blood-veſſels or of any two correſponding Blood-veſſels, and if the Numbers of their Pulſes in a given Time be inverſly as the Times of two Syſtoles of their Hearts; the Velocities of their Blood in two correſponding Blood-veſſels will be in the ſubduplicate Ratio of the Diameters of the Veſſels, that is, putting V, v *for the Velocities, and* D, d *for the Diameters of the Veſſels,* $V . v :: \surd D . \surd d$.

For the Velocities in any two Blood-veſſels, and conſequently in two correſponding Blood-veſſels, of the two Bodies, are as the Quantities of Blood which flow into thoſe Veſſels in two Syſtoles of their Hearts, apply'd to the Squares of the Diameters of the Veſſels and the Times of the two Syſtoles taken together; and the Quantities of Blood which flow into the Veſſels in two Syſtoles, are as the Capacities of the correſponding Ventricles of the Hearts, becauſe the Hearts are ſuppoſed to be free from the Influences of all diſturbing Cauſes, and the Bodies to be ſituated alike with reſpect to the Horizon; and therefore, putting L, l for the Lengths of two correſponding Blood-veſſels, or for the Lengths of the two Bodies, to which the Lengths of correſponding Blood-veſſels are ever proportional in perfectly well-proportion'd Bodies, and T, t for the Times of two Syſtoles, we ſhall have $V . v :: \frac{D^2 L}{D^2 T} . \frac{d^2 l}{d^2 t} :: \frac{L}{T} . \frac{l}{t}$: But by Suppoſition $P . p :: \frac{1}{T} . \frac{1}{t}$: And therefore $V . v :: LP . lp$.

By obſerving the Pulſes of healthful Bodies of different Lengths in the Morning when they were ſitting,

ting, I have found that the mean Numbers of Pulſes in a Minute of Bodies of any two different Lengths, each Mean being taken from the Pulſes of a Number of Bodies of each Length, are very nearly as $\frac{1}{L^{\frac{3}{4}}}$ and $\frac{1}{l^{\frac{3}{4}}}$: But the mean Numbers of Pulſes in a Minute of Bodies of thoſe two Lengths, are the true Pulſes of two healthful and perfectly well-proportion'd Bodies: And therefore $P \,.\, p :: \frac{1}{L^{\frac{3}{4}}} \,.\, \frac{1}{l^{\frac{3}{4}}}$.

And by comparing the Diameters of the Aorta of Bodies of different Lengths with the Lengths of the Bodies, as far as I can judge from the few Experiments I made, the Diameters are nearly in the ſubduplicate Ratios of the Lengths: But the mean Diameters of the Aorta of Bodies of two different Lengths, each Mean being taken from the

Diameters of this Veſſel of a Number of Bodies of each Length, are the true Diameters of the Aorta of two healthful and perfectly well-proportion'd Bodies of thoſe Lengths: And therefore $D \,.\, d :: \sqrt{L} \,.\, \sqrt{l}$, and $\sqrt{D} \,.\, \sqrt{d} :: L^{\frac{1}{4}} \,.\, l^{\frac{1}{4}}$.

And therefore in the Analogy $V \,.\, v :: LP \,.\, lp$, if inſtead of P and p, be ſubſtituted the Quantities $\frac{1}{L^{\frac{3}{4}}}$ and $\frac{1}{l^{\frac{3}{4}}}$ proportional to them, we ſhall have $V \,.\, v :: L^{\frac{1}{4}} \,.\, l^{\frac{1}{4}}$; and ſince $L^{\frac{1}{4}} \,.\, l^{\frac{1}{4}} :: \sqrt{D} \,.\, \sqrt{d}$, we ſhall have $V \,.\, v :: \sqrt{D} \,.\, \sqrt{d}$. Which was to be proved.

This *Propoſition* may be expreſs'd generally, by ſuppoſing $P \,.\, p :: \frac{1}{L^{m}} \,.\, \frac{1}{l^{m}}$, and $D \,.\, d :: L^{n} \,.\, l^{n}$. For then $V \,.\, v :: L^{1-m} \,.\, l^{1-m} :: D^{\frac{1-m}{n}} \,.\, d^{\frac{1-m}{n}}$. Hence, if hereafter it ſhall be found by a larger Experience, that the Meaſures of the Pulſes and Diameters of correſponding Blood-veſſels are different from thoſe aſſigned in this *Propoſition*, the Proportions of the Velocities in healthful and perfectly well-proportion'd Bodies of different Lengths, may be known.

For Inſtance, if it ſhall be found that m inſtead of being equal to $\frac{3}{4}$ is equal to $\frac{1}{2}$, and that n is $\frac{1}{2}$ as I have made it; then $V.v :: \surd L.\surd l :: D.d$. And if it ſhall be found, that m is $\frac{4}{5}$ and n $\frac{1}{2}$, then $V.v :: L^{\frac{1}{5}}.l^{\frac{1}{5}} :: D^{\frac{2}{5}}.d^{\frac{2}{5}}$. The Proportions of the Pulſes and Diameters of correſponding Blood-veſſels delivered from Experiments in this *Propoſition*, are, as far as I can judge, intirely agreeable to the Phænomena, and therefore I ſhall retain them till a larger Experience ſhall ſhew that they are not the Proportions which obtain in Nature.

I now come to conſider the Doctor's Remarks on my 24th *Propoſition*, which *Propoſition* ſtands thus.

THE Life of Animals is preſerved by acid Parts of the Air mixing with the Blood in the Lungs; which Parts diſſolve or attenuate the Blood, and preſerve its Heat, and by both theſe keep up the Motion of the Heart.

I proved this *Propoſition* from a Series of Experiments taken from Sir *Iſaac Newton*, Dr. *Hook*, Dr. *Lower*, and others. And the Manner in which I proved it was this.

From Experiments I proved *firſt*, that a conſtant Supply of freſh Air is neceſſary to preſerve the Life of Animals; *ſecondly*, that freſh Air preſerves Life in Animals by the very ſame Power, or by the Operation of the very ſame Parts, whereby it preſerves Fire and Flame in ſulphureous and unctuous Subſtances when once they are kindled; and *thirdly*, that Air preſerves Fire and Flame in ſulphureous and unctuous Subſtances when once they are kindled, and conſequently the Life of Animals, by its acid Particles. Had this Author attended to the Experiments from which I proved the ſecond and third Particulars, he could not but have ſeen that they were juſtly proved. For, ſince Animals die in Air rendered effete by burning Coals or Candles in it till they are extinguiſhed, and glowing Coals or Candles are extinguiſhed in Air rendered effete by Animals breathing in it till they die; it clearly follows, that Air preſerves Fire and Flame, and the Life of Animals, by the ſame Power, or by the Operation of the ſame Parts: And, ſince Fire and Flame can be produced without Air by mixing compound Spirit of Nitre, which is an Acid, in a certain Proportion with ſome Oils *in vacuo*, and can be preſerved without Air in a Mixture of common Sulphur and Nitre powdered when once it is kindled, it follows that Air preſerves Fire and Flame in ſulphureous and unctuous Subſtances when once they are kindled, and the Life of Animals, by means of its acid Particles. There is no way of proving this Inference to be falſe, but by proving that there are no acid Particles in the Air. But the Air abounds with ſuch Particles, as appears from the Nitre, which is found ſticking to the Sides of plaiſtered Walls, and to the Mortar between the Bricks of Brick Walls, which are defended from the Rain which would diſſolve it, and from the Sun which would rarefy it and cauſe it to be exhaled into the Air. For Sir *Iſaac Newton* has proved from Experiments, that Salts are compoſed of Acid and Earth united by Attraction: Whence it follows, that the Formation of Nitre on the Sides of plaiſtered Walls and on the Mortar of Brick Walls defended from Rain and Sun, muſt be owing to acid Parts of the Air uniting with the earthy alcalious Parts of the Lime, by the ſtrong Attraction which intercedes them: And therefore the Air abounds with acid Particles.

The third Particular proved by Experiments entirely agrees with Sir *Iſaac Newton*'s Reaſoning from the ſame Experiments, which I ſhall ſet down in his own Words. "Alſo "ſome ſulphureous Steams, at all "Times when the Earth is dry, "aſcending into the Air, ferment

" there with nitrous Acids, and
" ſometimes taking Fire, cauſe
" Lightning and Thunder and
" fiery Meteors. For the Air abounds
" with acid Vapours fit to
" promote Fermentations, as appears
" by the ruſting of Iron and
" Copper in it, the kindling of
" Fire by blowing, and the beating
" of the Heart by means of
" Reſpiration." *Opt. p.* 355.-----
" And the Cauſe of Fermentation,
" *which is an Acid*, by which the
" Heart and Blood of Animals are
" kept in perpetual Motion and
" Heat." *Opt. p.* 375.

But this Author ſays, p. 75. *'Tis well known, that all Fluids, Acids as well as others, excepting Oil, will extinguiſh Fire, and that there is nothing in Nature but Oil that Fire can feed upon; for when the volatile and fixed Oil of any combuſtible Subſtance is conſumed and evaporated, the Fire* *can act no longer upon what remains, how much ſoever it may be aſſiſted and ſupply'd with Air: Acids will check and extinguiſh Fire ſooner than common Water, and any acid Vapour in the Air is more ſuffocating and deſtructive than any common watry Vapour and Fume.* I grant that Fire and Flame cannot ſubſiſt without oily and ſulphureous Particles, neither can they ſubſiſt without an Acid; for there was an Acid in all the Mixtures of the Experiments under the third Head: So that by thoſe Experiments both an Acid and an Oil are neceſſary to the Production and Preſervation of Fire and Flame. But to go on, he ſays, *that all Fluids, Acids as well as others, except Oil, will extinguiſh Fire.---And that Acids will check and extinguiſh Fire ſooner than common Water.* And what then? will it from thence follow, that a nitrous Acid cannot when mix'd with ſome Oils in a certain Proportion produce Fire and Flame? By no means. For it has been found by Experiments, that if two Parts of compound Spirit of Nitre be poured on one Part of Oil of Cloves or Caraway Seeds, or of any ponderous Oil of vegetable or animal Subſtances, and particularly Oil of human Blood, or Oil of Turpentine thickened with a little Balſam of Sulphur, the Liquors grow ſo very hot in mixing, as preſently to ſend up a burning Flame. *Opt. Newt. p.* 353. *Phil. Tranſ. N.* 213. *p.* 200. And therefore, this acid Spirit muſt be allow'd to have a Power of producing Heat and Flame when mixed with certain oily and ſulphureous Subſtances in a certain Proportion: And ſo likewiſe may the nitrous Acid of the Air have a Power of preſerving Fire and Flame in ſulphureous and unctuous Subſtances when kindled, and of keeping

ing up a gentle Heat in the Blood of Animals by mixing with it in the Lungs, and fermenting with its oily Parts; and that in fact it does ſo, the Experiments by me alledged have fully proved.

And I am ſtill farther convinced of the Neceſſity of allowing an Acid in the Air to preſerve the Life of Animals, when I conſider the Inſufficiency of all other Accounts of *Reſpiration*. As to the Uſe aſſigned to the Air by this Gentleman, namely, *that it ſerves as a proper exhaling Medium to receive and carry off thoſe copious Diſcharges of humid Effluvia or moiſt Vapour, which all living Creatures, and all combuſtible Matter under the Action of Fire, are inceſſantly emitting and throwing out*; I muſt beg leave to tell him, that this is an old trite Hypotheſis, without any the leaſt Foundation from Reaſon and Experiments. For Vapours and Exhalations are not thrown off from humid Bodies by any Virtue in the Air, but by the repulſive Powers of their Particles, when by the Action of Heat, they are once ſeparated from the Bodies, and are got beyond the Spheres of their Attractions and of the Attractions of one another. For this repulſive Power will carry off the ſeparated Particles, as well *in vacuo* as in the open Air.

I could have wiſh'd Dr. *Morgan* had conſidered my *Animal Oeconomy* with a little more Temper as well as Care, and then I am ſatisfied he would have ſaved both himſelf and me ſome Trouble. It is however ſome Satisfaction to me, that he has given me an Opportunity of publickly declaring my ſelf,

Sir, Your moſt Obliged,
July 12. 1735. *Humble Servant,*

BRYAN ROBINSON.

FINIS.

ERRATA.

P. 10. *l.* 5. *for* If A *r.* If a. *p.* 16. *l.* 20. *and p.* 17. *l.* 2. *for* $\frac{Q}{.7853816D^2T}$ *r.* $\frac{Q}{.785398 16D^2T}$ *ibid. for* $67.97239\sqrt{\frac{A^2H}{A^2-a^2}}$ *r.* $53.3854 D^2T \sqrt{\frac{A^2H}{A^2-a^2}}$, *and for* $W=53.38426D^2T\sqrt{\frac{A^2H}{A^2-a^2}}$ *r.* $W=28.1756 D^2T \sqrt{\frac{A^2H}{A^2-a^2}}$. *p.* 32. *l.* 8. *for* inſide *r.* Side. *ibid. l.* 11. *for* Water. *r.* Water: And therefore the Force generating the Motion of Water flowing through a Pipe is equal to the Weight of a Cylinder of Water whoſe Magnitude is $a \times \frac{A^2H}{A^2-a^2}$, and is proportional to $D^2 \times \frac{A^2H}{A^2-a^2}$. However I will grant that F is ever as D^2H, and accordingly conſider the Conſequences he draws from it. *p.* 33. *l.* 3. *del.* ing. *p.* 50. *l.* 12. *for* aſcalious *r.* alcalious.